

Spatial Reference Systems

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1 *Introduction*

Historic Development
Digital Technologies






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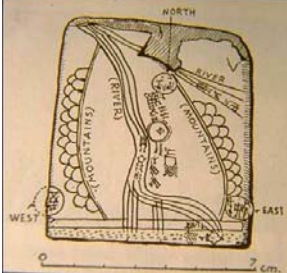
Historic Development:
Antiquity

- Cartography in ancient civilizations (Babylonians, Egyptians, Chinese, Greeks, Romans)
- Only few representations survived
- Much theory development by Greeks
- Little innovation by the Romans




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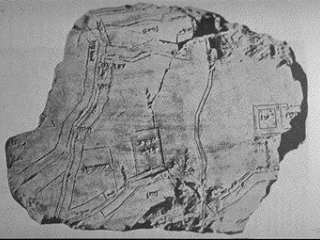

Ga-Sur Tablet (3800 B.C.)




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City Map of Nippur (1500 B.C.)

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Historic Development:
Antiquity

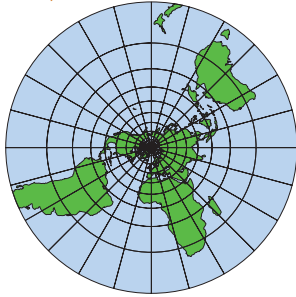
- Determination of the size of the Earth by *Eratosthenes* (276-195 B.C.)
- *Hipparchus* (190-125 B.C.): division of equator into 360 degrees; development of stereographic and orthographic projections
- *Marinus of Tyre* (70 – 130 A.D.) rectangular grid of latitude and longitude
- *Ptolemy* (87-150 A.D.): conic projection

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*Stereographic Projection
(Hipparchus)*

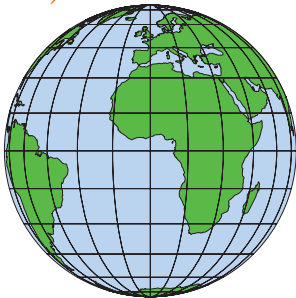


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*Orthographic Projection
(Hipparchus)*

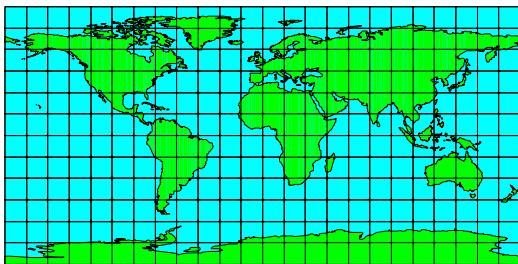


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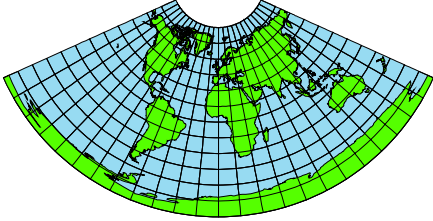
*Rectangular Projection
(Marinus of Tyre)*



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Equidistant Projection (Ptolemy)






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Historic Development:
Medieval Ages

- Stagnation in Europe
- The works of antiquity were rejected in Europe but preserved by Arab scientists

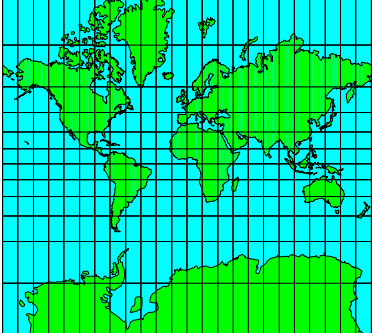
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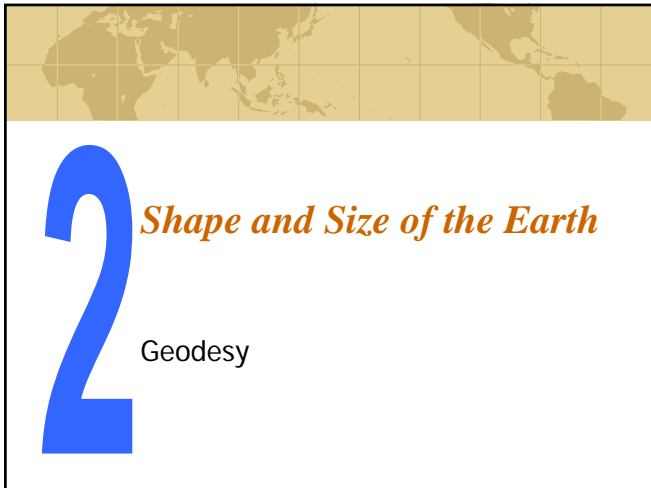
Historic Development: Modern Times

- Need for navigation maps
- World map of *Gerhard Mercator* (1512-1594) published in 1569
- Ellipsoidal shape of the Earth
- Theory of distortion (*Tissot*)
- Topographic map projections (Gauß-Krüger, UTM)
- GIS, GPS

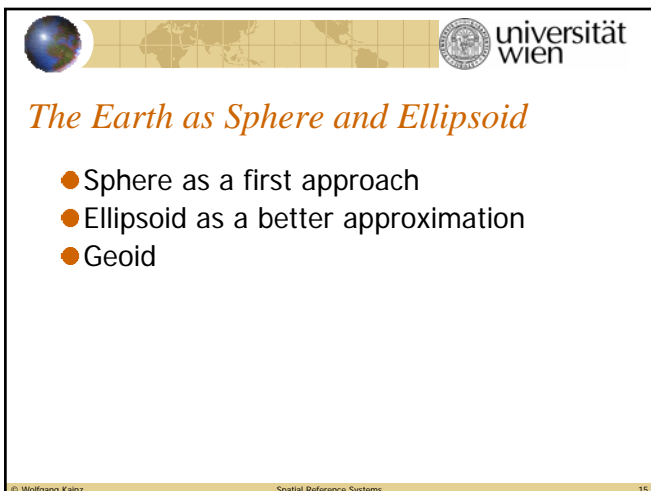
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The slide features a header with a globe icon, a world map, and the University of Vienna logo. The title "Mercator Projection" is written in orange. Below it is a world map with a blue grid representing the Mercator projection. The footer contains the text "© Wolfgang Kainz Spatial Reference Systems 14".



The slide has a header with a world map. A large blue number "2" is on the left, followed by the title "Shape and Size of the Earth" in orange. Below the title is the word "Geodesy".



The slide features a header with a globe icon, a world map, and the University of Vienna logo. The title "The Earth as Sphere and Ellipsoid" is in orange. Below it is a bulleted list: "● Sphere as a first approach", "● Ellipsoid as a better approximation", and "● Geoid". The footer contains the text "© Wolfgang Kainz Spatial Reference Systems 15".

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Geodesy according to Eratosthenes (276 – 195 B.C.)

sun

N

S

Alexandria

Syene (Assuan)

$2R\pi = 360$

$b = \gamma$

$R = \frac{180 \cdot b}{\pi \cdot \gamma}$

$b = 742.5 \text{ km}$

$\gamma = 7'12''$

$R = 5909 \text{ km}$

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Ellipsoids (spheroids)

Spheroid	Year	Semi Major Axis	Semi Minor Axis	Flattening	Applications
Everest	1830	6 377 276 m	6 356 075 m	1 : 300,80	India, Pakistan, Nepal, Sri Lanka
Bessel	1841	6 377 397 m	6 356 079 m	1 : 299,15	Central Europe, Asia
Airy	1849	6 377 563 m	6 356 257 m	1 : 299,33	Great Britain
Clarke	1866	6 378 206 m	6 356 584 m	1 : 294,98	North America
Clarke	1880	6 378 249 m	6 356 515 m	1 : 293,47	Africa, France
Hayford = International (1924)	1909	6 378 388 m	6 356 912 m	1 : 297,00	World except North America
Krassowskij	1940	6 378 245 m	6 356 863 m	1 : 298,30	Russia, Central Asia, Antarctica, China
IUGG (GRS 80)	1980	6 378 137 m	6 356 752 m	1 : 298,26	North America (NAD 83)
WGS	1984	6 378 137 m	6 356 752 m	1 : 298,26	GPS

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Geoid

- Equipotential surface of the Earth's gravity field
- The direction of gravity is everywhere perpendicular to the surface
- The geoid has tides

Source: Lexikon der Kartographie und Geomatik, Spektrum Akademischer Verlag GmbH, Heidelberg, Berlin, 2002

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Sphere as Reference Surface

- Calculations on the sphere are simpler than on the spheroid
- Suitable for scales 1 : 2 000 000 and smaller
- The radius of the sphere is approximately 6 370 km.

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Computation of Radius of Sphere (Example Bessel)

$$V_K = \frac{4}{3} \pi r^3$$
$$V_E = \frac{4}{3} \pi a^2 b$$
$$r_V = \sqrt[3]{a^2 b}$$
$$r_V = 6370283 \text{ m}$$

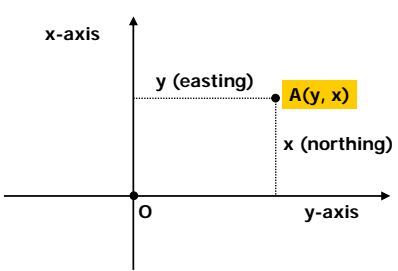
Equal volume with
ellipsoid

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3 *Reference Systems*

Coordinate systems
Positioning
Datums

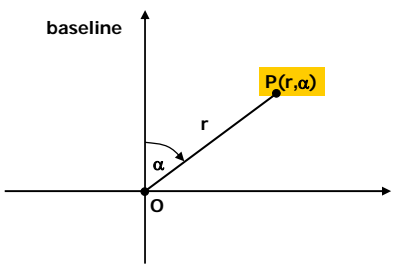


The diagram shows a 2D Cartesian coordinate system with a vertical **x-axis** and a horizontal **y-axis**. The origin is labeled **O**. A point **A(y, x)** is plotted in the first quadrant. Dotted lines indicate the coordinates: a horizontal line from the **y-axis** to the point is labeled **y (easting)**, and a vertical line from the point to the **x-axis** is labeled **x (northing)**.

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Cartesian Coordinate System

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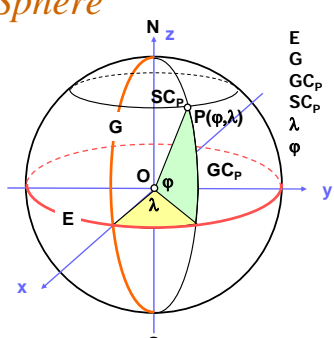


The diagram shows a 2D polar coordinate system with a vertical **baseline** and a horizontal axis. The origin is labeled **O**. A point **P(r, α)** is plotted in the first quadrant. A line segment of length **r** connects the origin to the point, and the angle between the baseline and this segment is labeled **α**.

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Polar Coordinate System

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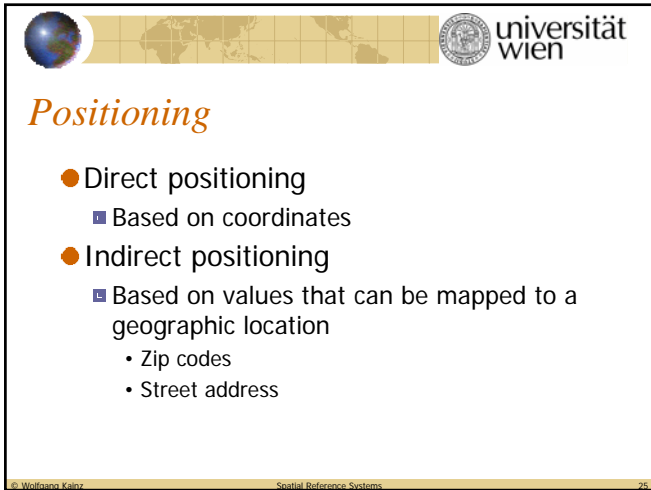
The diagram shows a sphere with a vertical **z-axis** (North **N** to South **S**) and a horizontal **y-axis**. The origin is **O**. The equator is labeled **E**. A point **P(φ, λ)** is on the sphere. A great circle through **P** is labeled **GC_P**, and a small circle through **P** is labeled **SC_P**. The angle between the **z-axis** and the radius to **P** is **φ** (latitude). The angle between the zero meridian **G** and the great circle **GC_P** is **λ** (longitude).

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Geographic Coordinates on the Sphere

- E equator
- G zero meridian
- GC_P great circle through P
- SC_P small circle through P
- λ longitude of P
- φ latitude of P

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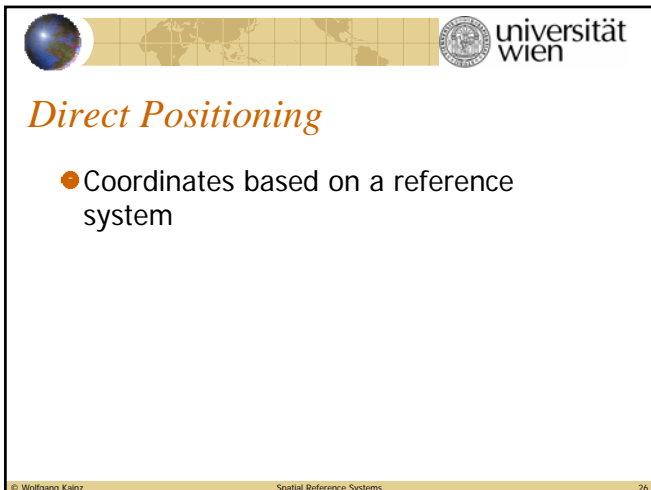


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Positioning

- Direct positioning
 - Based on coordinates
- Indirect positioning
 - Based on values that can be mapped to a geographic location
 - Zip codes
 - Street address

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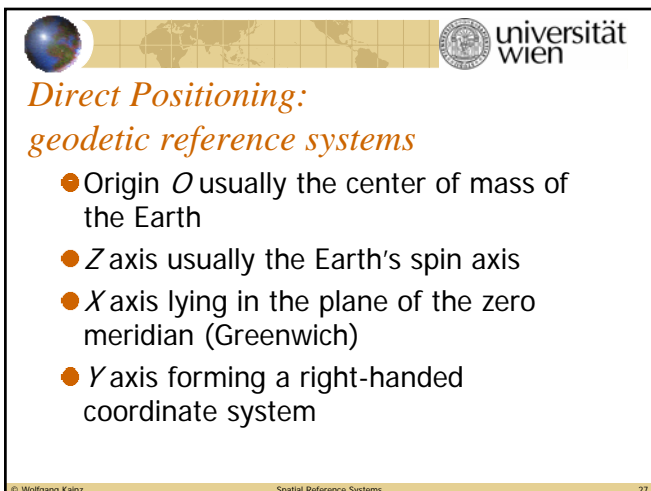


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Direct Positioning

- Coordinates based on a reference system

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Direct Positioning: geodetic reference systems

- Origin O usually the center of mass of the Earth
- Z axis usually the Earth's spin axis
- X axis lying in the plane of the zero meridian (Greenwich)
- Y axis forming a right-handed coordinate system

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Reference System

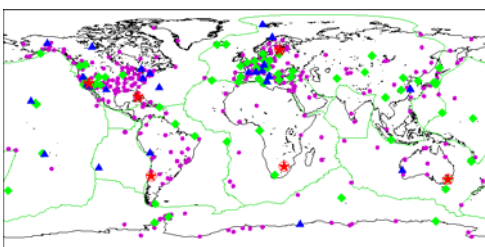
ITRS (International Terrestrial Reference System) of IERS (International Earth Rotation Service) [since 1.1.1988], hpiers.obspm.fr

- realized by reference frames
 - International Terrestrial Reference Frame (www.ensg.ign.fr/ITRF/)
 - ITRF2000

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ITRF2000 Locations



1 2 3 4
Collocated techniques: 70 25 6


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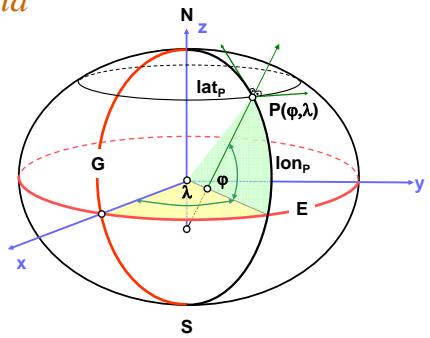
Geodetic Ellipsoid

- Representation of the geoid by an ellipsoid of rotation
 - Center coincides with the origin O of the reference system
 - Semi-minor axis coincides with Z
 - X -axis pierces the ellipsoid at latitude 0 (equator) and longitude 0 (zero meridian)


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Geographical Coordinates on the Ellipsoid




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Geocentric Reference System

- World Geodetic System 84 (WGS84)
 - Basically identical to the GRS 80 reference ellipsoid
 - used for the Global Positioning System (GPS)




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Geodetic Datum

- 3-dimensional geodetic datum
 - Definition of a geodetic ellipsoid
 - Location of the origin of ellipsoid relative to the center of mass of the Earth
 - Orientation of the Z-axis relative to the Earth rotation axis
 - Position of X-axis to the zero meridian
 - Y-axis is added to a right-handed system
- 2-dimensional geodetic datum
 - Position of a 2-dimensional coordinate system to the Earth body
 - Main point as origin of datum
 - Geoid height at origin
 - Parameters of geodetic ellipsoid

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Geodetic Datum: examples




- 2-dimensional datum (Austria)
 - MGI (Bessel ellipsoid, Gauß-Krüger projection)
- 3-dimensional datum (Austria)
 - WGS 84 (GRS80, UTM)

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4 *Map Projections*

Mathematical mapping
Reference plane and aspect






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Map Projections

Mapping of the surface of the ellipsoid (or sphere) to a plane in Cartesian or polar coordinates

$x = f_1(\varphi, \lambda)$ $r = f_3(\varphi, \lambda)$
 $y = f_2(\varphi, \lambda)$ $\alpha = f_4(\varphi, \lambda)$




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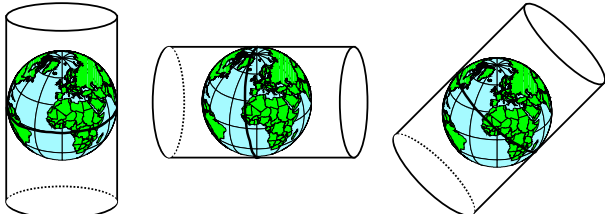
Characteristics of Map Projections

- Conformal
- Equal area
- Equidistant

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


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Cylindrical Projections

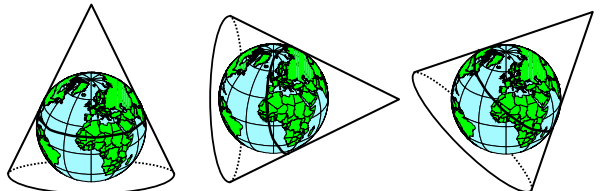


normal aspect transverse aspect oblique aspect

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Conic Projections

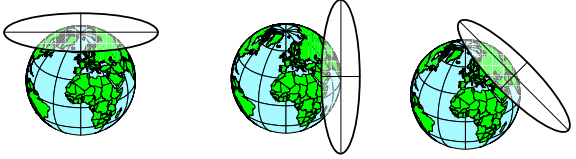


normal aspect transverse aspect oblique aspect

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Azimuthal Projections



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Map Projections

A: polyconic **B**

$y = f_1(\varphi, \lambda)$ $y = f_1(\lambda)$

$x = f_2(\varphi, \lambda)$ $x = f_2(\varphi, \lambda)$

C: pseudocylindrical **D: conic**

$y = f_1(\varphi, \lambda)$ $y = f_1(\lambda)$

$x = f_2(\varphi)$ $x = f_2(\varphi)$

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Map Projections (polar coordinates)

A: polyconic **B**

$r = f_1(\varphi, \lambda)$ $r = f_1(\varphi, \lambda)$

$\theta = f_2(\varphi, \lambda)$ $\theta = f_2(\lambda)$

C: pseudoazimuthal **D: azimuthal**

$r = f_1(\varphi)$ $r = f_1(\varphi)$

$\theta = f_2(\varphi, \lambda)$ $\theta = f_2(\lambda)$

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