Spatial Reference Systems

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1 Introduction

Historic Development
Digital Technologies

Historic Development: Antiquity

- Cartography in ancient civilizations (Babylonians, Egyptians, Chinese, Greeks, Romans)
- Only few representations survived
- Much theory development by Greeks
- Little innovation by the Romans

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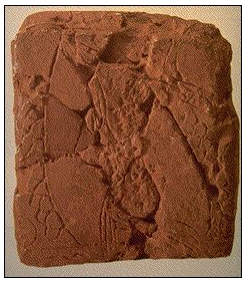
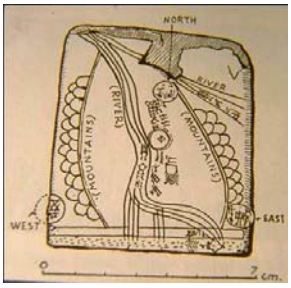
Ga-Sur Tablet (3800 B.C.)





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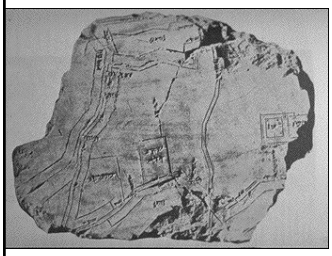
City Map of Nippur (1500 B.C.)



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Historic Development: Antiquity

- Determination of the size of the Earth by *Eratosthenes* (276-195 B.C.)
- *Hipparchus* (190-125 B.C.): division of equator into 360 degrees; development of stereographic and orthographic projections
- *Marinus of Tyre* (70 – 130 A.D.) rectangular grid of latitude and longitude
- *Ptolemy* (87-150 A.D.): conic projection

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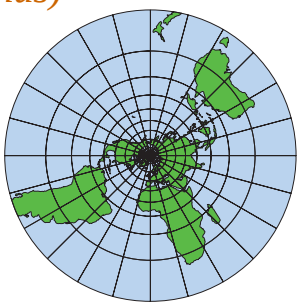
Stereographic Projection (Hipparchus)



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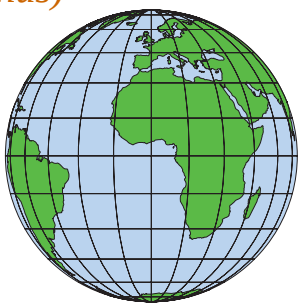
Orthographic Projection (Hipparchus)



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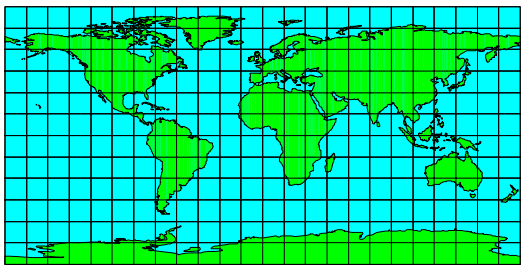
Rectangular Projection (Marinus of Tyre)



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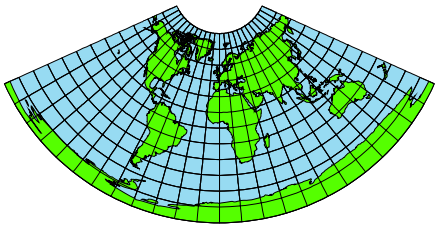
Equidistant Projection (Ptolemy)



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Historic Development: Medieval Ages

- Stagnation in Europe
- The works of antiquity were rejected in Europe but preserved by Arab scientists

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Historic Development: Modern Times

- Need for navigation maps
- World map of *Gerhard Mercator* (1512-1594) published in 1569
- Ellipsoidal shape of the Earth
- Theory of distortion (*Tissot*)
- Topographic map projections (Gauß-Krüger, UTM)
- GIS, GPS

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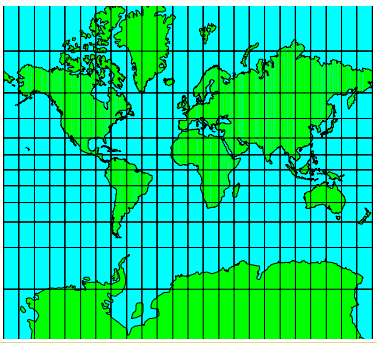
Mercator Projection



2 *Shape and Size of the Earth*

Geodesy

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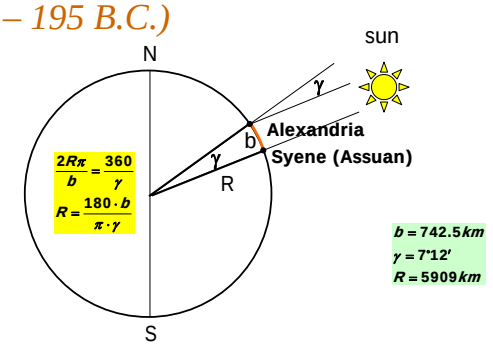
The Earth as Sphere and Ellipsoid

- Sphere as a first approach
- Ellipsoid as a better approximation
- Geoid

Geodesy according to Eratosthenes (276 – 195 B.C.)

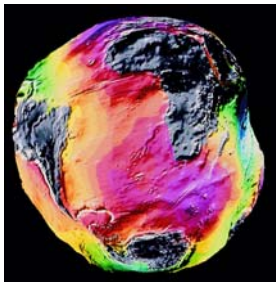


Ellipsoids (spheroids)

Spheroid	Year	Semi Major Axis	Semi Minor Axis	Flattening	Applications
Everest	1830	6 377 276 m	6 356 075 m	1 : 300,80	India, Pakistan, Nepal, Sri Lanka
Bessel	1841	6 377 397 m	6 356 079 m	1 : 299,15	Central Europe, Asia
Airy	1849	6 377 563 m	6 356 257 m	1 : 299,33	Great Britain
Clarke	1866	6 378 206 m	6 356 584 m	1 : 294,98	North America
Clarke	1880	6 378 249 m	6 356 515 m	1 : 293,47	Africa, France
Hayford = International (1924)	1909	6 378 388 m	6 356 912 m	1 : 297,00	World except North America
Krassowskij	1940	6 378 245 m	6 356 863 m	1 : 298,30	Russia, Central Asia, Antarctica, China
IUGG (GRS 80)	1980	6 378 137 m	6 356 752 m	1 : 298,26	North America (NAD 83)
WGS	1984	6 378 137 m	6 356 752 m	1 : 298,26	GPS

Geoid

- Equipotential surface of the Earth's gravity field
- The direction of gravity is everywhere perpendicular to the surface
- The geoid has tides



Source: Lexikon der Kartographie und Geomatik, Spektrum Akademischer Verlag GmbH, Heidelberg, Berlin, 2002

Sphere as Reference Surface

- Calculations on the sphere are simpler than on the spheroid
- Suitabel for scales 1 : 2 000 000 and smaller
- The radius of the sphere is approximately 6 370 km.

Computation of Radius of Sphere (Example Bessel)

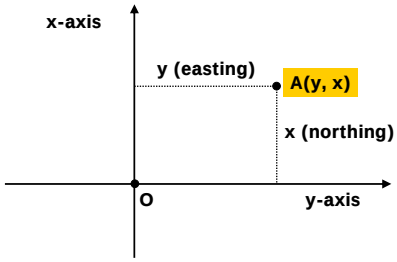
$$V_K = \frac{4}{3}\pi r^3$$
$$V_E = \frac{4}{3}\pi a^2 b$$
$$r_V = \sqrt[3]{a^2 b}$$
$$r_V = 6370283\,\text{m}$$

Equal volume with ellipsoid

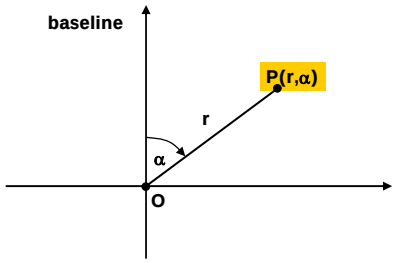
3 Reference Systems

Coordinate systems
Positioning
Datums

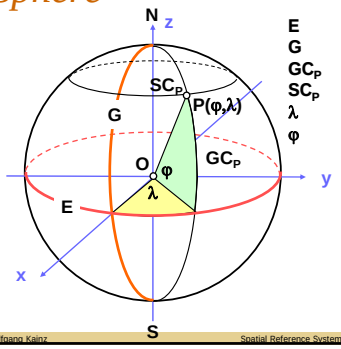
Cartesian Coordinate System



Polar Coordinate System



Geographic Coordinates on the Sphere



E	equator
G	zero meridian
GC_P	great circle through P
SC_P	small circle through P
λ	longitude of P
φ	latitude of P

Positioning

- Direct positioning
  - Based on coordinates
- Indirect positioning
  - Based on values that can be mapped to a geographic location
    - Zip codes
    - Street address

Direct Positioning

- Coordinates based on a reference system

Direct Positioning: geodetic reference systems

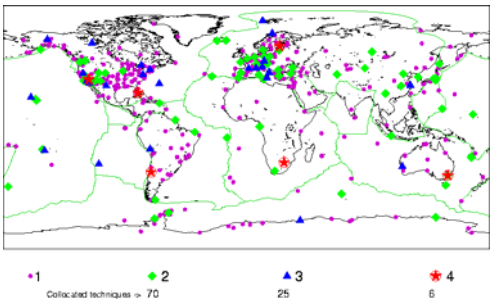
- Origin *O* usually the center of mass of the Earth
- *Z* axis usually the Earth's spin axis
- *X* axis lying in the plane of the zero meridian (Greenwich)
- *Y* axis forming a right-handed coordinate system

Reference System

ITRS (International Terrestrial Reference System) of IERS (International Earth Rotation Service) [since 1.1.1988], hpiers.obspm.fr

- realized by reference frames
  - International Terrestrial Reference Frame (www.ensg.ign.fr/ITRF/)
  - ITRF2000

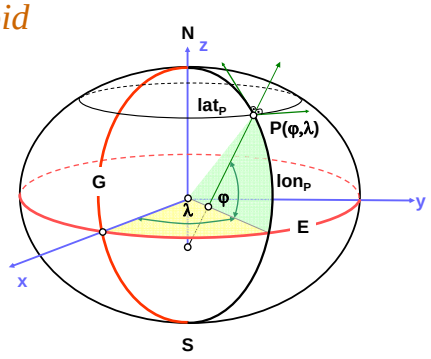
ITRF2000 Locations



Geodetic Ellipsoid

- Representation of the geoid by an ellipsoid of rotation
  - Center coincides with the origin *O* of the reference system
  - Semi-minor axis coincides with *Z*
  - *X*-axis pierces the ellipsoid at latitude 0 (equator) and longitude 0 (zero meridian)

Geographical Coordinates on the Ellipsoid



Geocentric Reference System

- World Geodetic System 84 (WGS84)
  - Basically identical to the GRS 80 reference ellipsoid
  - used for the Global Positioning System (GPS)

Geodetic Datum

- 3-dimensional geodetic datum
  - Definition of a geodetic ellipsoid
  - Location of the origin of ellipsoid relative to the center of mass of the Earth
  - Orientation of the Z-axis relative to the Earth rotation axis
  - Position of X-axis to the zero meridian
  - Y-axis is added to a right-handed system
- 2-dimensional geodetic datum
  - Position of a 2-dimensional coordinate system to the Earth body
    - Main point as origin of datum
    - Geoid height at origin
    - Parameters of geodetic ellipsoid

Geodetic Datum: examples

- 2-dimensional datum (Austria)
  - MGI (Bessel ellipsoid, Gauß-Krüger projection)
- 3-dimensional datum (Austria)
  - WGS 84 (GRS80, UTM)

4 Map Projections

Mathematical mapping
Reference plane and aspect

Map Projections

Mapping of the surface of the ellipsoid (or sphere) to a plane in Cartesian or polar coordinates

$$x = f_1(\varphi, \lambda) \qquad r = f_3(\varphi, \lambda)$$
$$y = f_2(\varphi, \lambda) \qquad \alpha = f_4(\varphi, \lambda)$$

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Characteristics of Map Projections

- Conformal
- Equal area
- Equidistant

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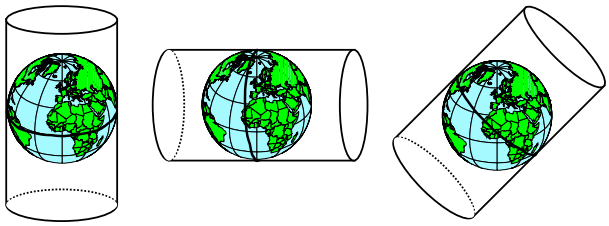
Cylindrical Projections



normal aspect transverse aspect oblique aspect

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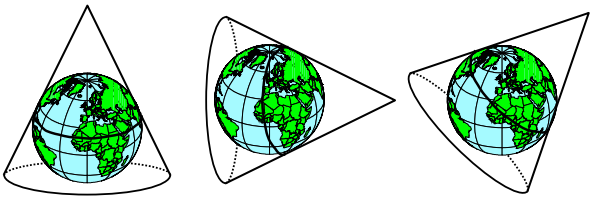
Conic Projections



normal aspect transverse aspect oblique aspect

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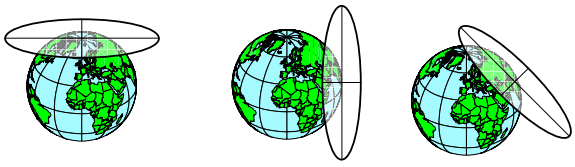
Azimuthal Projections



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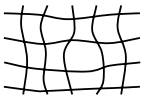
Map Projections

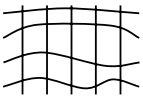
A: polyconic

$y = f_1(\varphi, \lambda)$
$x = f_2(\varphi, \lambda)$



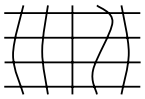
B

$y = f_1(\lambda)$
$x = f_2(\varphi, \lambda)$



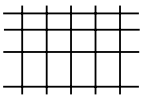
C: pseudocylindrical

$y = f_1(\varphi, \lambda)$
$x = f_2(\varphi)$



D: conic

$y = f_1(\lambda)$
$x = f_2(\varphi)$



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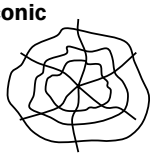
Map Projections (polar coordinates)

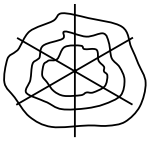
A: polyconic

$r = f_1(\varphi, \lambda)$
$\theta = f_2(\varphi, \lambda)$



B

$r = f_1(\varphi, \lambda)$
$\theta = f_2(\lambda)$



C: pseudoazimuthal

$r = f_1(\varphi)$
$\theta = f_2(\varphi, \lambda)$

D: azimuthal

$r = f_1(\varphi)$
$\theta = f_2(\lambda)$